

63-3-2

SSD-TDR-63-39

REPORT NO.  
TDR-169(3240-10)TR

CATALOGED BY ACTIA  
AS AD NO. 401225

## Heat Transfer in Power Transistors

401 225

15 FEBRUARY 1963

*Prepared by W. R. WILCOX  
Materials Sciences Laboratory*

*Prepared for* COMMANDER SPACE SYSTEMS DIVISION

UNITED STATES AIR FORCE

*Inglewood, California*

ASTIA  
RECEIVED  
APR 1 1963  
JULIEN 7 1963  
TISIA A



LABORATORIES DIVISION • AEROSPACE CORPORATION  
CONTRACT NO. AF 04(695)-169

SSD-TDR-63-39

Report No.  
TDR-169(3240-10)TR-3

HEAT TRANSFER IN POWER TRANSISTORS

Prepared by

W. R. Wilcox  
Materials Sciences Laboratory

AEROSPACE CORPORATION  
El Segundo, California

Contract No. AF 04(695)-169

15 February 1963

Prepared for

COMMANDER SPACE SYSTEMS DIVISION  
UNITED STATES AIR FORCE  
Inglewood, California

### ABSTRACT

The internal heat transfer problem for a typical power-transistor structure has been solved analytically. The relations among current distribution, heat generation, and temperature distribution have been derived. Usage of the resulting equations is illustrated by application to the most elementary problem, namely, uniform heat generation under the emitter.

## CONTENTS

I.	INTRODUCTION . . . . .	1
II.	THEORETICAL ANALYSIS . . . . .	1
	A. BASIC SOLUTION . . . . .	1
	B. RELATION TO EMITTER TEMPERATURE . . . . .	5
	C. RELATION BETWEEN CURRENT DISTRIBUTION AND HEAT GENERATION . . . . .	6
III.	EXAMPLES. . . . .	7
	A. SEMI-INFINITE ONE-DIMENSIONAL TRANSISTOR . . . . .	7
	B. UNIFORM HEAT GENERATION . . . . .	8
	C. EMITTER PARTLY EFFECTIVE . . . . .	15
	D. OTHER HEAT DISTRIBUTIONS. . . . .	17
IV.	CONCLUSIONS. . . . .	17
	NOMENCLATURE . . . . .	19
	ACKNOWLEDGEMENT . . . . .	23

## FIGURES

1	Structure of a Typical Power Transistor . . . . .	2
2	Geometry and Coordinate System for Analysis of Heat Transfer in Power Transistor of Fig. 1 . . . . .	2
3	Maximum Temperature in Power Transistor as a Function of the Ratio of Device Thickness to Half of Emitter Spacing . . . . .	12
4	Maximum Temperature in Power Transistor as a Function of the Ratio of Emitter Width to Emitter Spacing . . . . .	14
5	Maximum Temperature in Power Transistor as a Function of the Ratio of Half of Emitter Spacing to Device Thickness . . . . .	16

## I. INTRODUCTION

One limiting factor in the operation of a transistor is the internal heating due to the flow of current. Not only does heating affect the characteristics of the transistor, but in excessive amounts it may also cause certain junction areas to become nearly intrinsic. When this happens, the transistor action ceases altogether. In power transistors, which are intended to carry large current loads, the internal heating is an important design consideration. In this paper, the heat transfer characteristics of a typical power transistor are examined theoretically.

## II. THEORETICAL ANALYSIS

### A. BASIC SOLUTION

A diagram of one type of power transistor structure is shown in Fig. 1. On the top of the transistor is a parallel series of long emitters. The emitters and the base region are actually much thinner than shown in the side section. The collector is securely attached to a heat sink. Most of the heat generated in the structure is developed in the high-resistivity region near the collector-base junction. Virtually all of the heat generated in the transistor is removed by the heat sink. The problem is to derive an expression for the temperature distribution inside the structure in order to locate any hot spots in the junction regions.

It was expected that, because of end effects, the hottest portion of each emitter was half-way down its length (top view in Fig. 1); hence, we solved for this area only. A cross-section of a single emitter was examined, using two of several possible techniques. One of the techniques used was to solve for a single heat source on one side of infinite parallel planes (thus neglecting side effects). In this case, the total solution for several emitters would be obtained merely by superposing the individual solutions. Unfortunately, however, we were unable to obtain the desired result by using this technique.

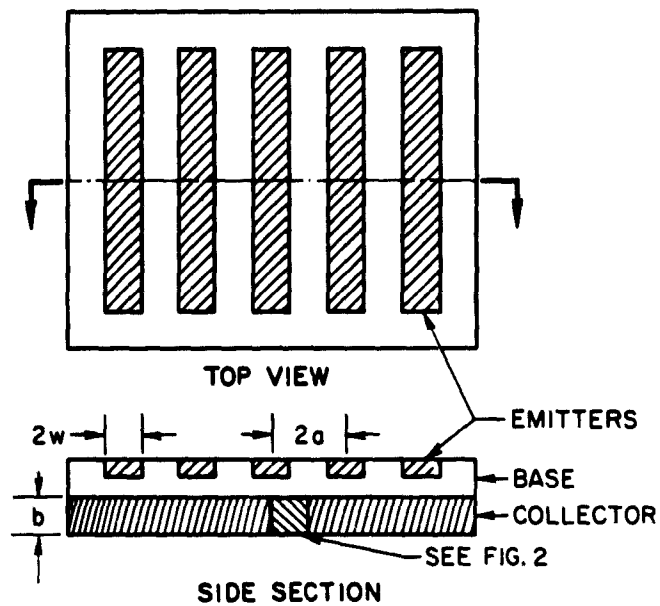


Fig. 1. Structure of Typical Power Transistor  
Collector is actually very much  
thicker than base plus emitter.

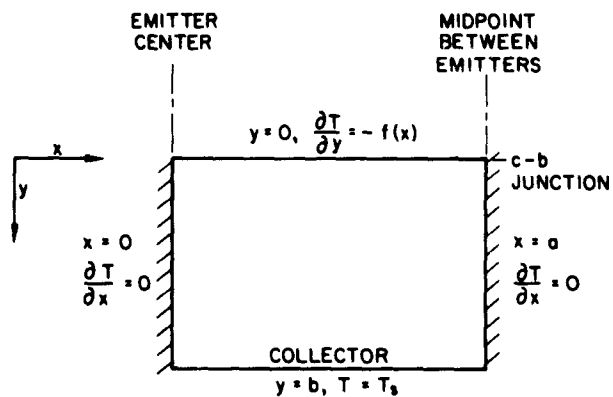


Fig. 2. Geometry and Coordinate System for Analysis of  
Heat Transfer in Power Transistor of Fig. 1

As an alternate approach, we considered the central emitter in a large collection. The central emitter was expected to be the warmest; hence, this was the one of most interest. By considering the central emitter, we again neglected side effects; however, with this technique, the final result should still be quite good even if the total number of emitters is as small as five.

A model of the problem under consideration is shown in Fig. 2. Because of symmetry, no heat flows across the dividing plane half-way between the emitters and no heat flows from one side of an emitter to another. Hence, these two boundaries are considered to be insulated. The heat sink, on the bottom of the rectangle at  $y = b$ , is at temperature  $T_s$ . Half of the collector-base junction beneath an emitter occupies the upper left portion of the rectangle at  $y = 0$ .

The assumptions made in this analysis are summarized as follows:

- 1) homogeneous and isotropic thermal conductivity
- 2) steady state conditions
- 3) infinite number of infinitely long emitters, i. e., end effects are neglected
- 4) heat is generated only along the collector-base junction,  $y = 0$ . (This primarily requires very thin emitter and base regions.)
- 5) temperature at  $y = b$  is uniform at  $T_s$

With these assumptions, the differential equation governing this process is\*

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (1)$$

---

\* Definitions of symbols are given in the Nomenclature at the end of this report.

and the boundary conditions are:

$$\text{at } x = 0, \frac{\partial T}{\partial x} = 0$$

$$\text{at } x = a, \frac{\partial T}{\partial x} = 0$$

$$\text{at } y = 0, \frac{\partial T}{\partial y} = -f(x)$$

$$\text{at } y = b, T = T_s$$

It may easily be shown that the following equation satisfies the above conditions, and is thus the general solution to our problem.

$$T - T_s = \frac{a_0}{2}(b - y) + \sum_{n=1}^{\infty} a_n \left( \frac{a}{n\pi} \right) \cos \frac{n\pi x}{a} \sinh \frac{(b - y)n\pi}{a} \operatorname{sech} \frac{bn\pi}{a} \quad (2)$$

The constants  $a_n$  are found from the boundary condition at  $y = 0$ ,

$$\frac{\partial T}{\partial y} = -\frac{a_0}{2} - \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{a} = -f(x) \quad (3)$$

This forms a Fourier series and hence

$$a_n = \frac{2}{a} \int_0^a f(x) \cos \frac{n\pi x}{a} dx \quad (n = 0, 1, 2, \dots) \quad (4)$$

This solution could also be applied to the whole device, as shown in Fig. 1, rather than to a single emitter. This is true because the device is generally much wider than it is thick so that the edges may be considered to be



insulated. The only alteration in the solution would be in the definition of  $a$ , which would then become the total device width. The heat generation  $f(x)$  would in this case be periodic with the emitter spacing. A treatment of this type would also take into account the side effects of a finite number of emitters.

#### B. RELATION TO EMITTER TEMPERATURE

From an experimental standpoint, temperature is much more easily measured than heat flux. Therefore, it is of interest to have a solution for the problem with the temperature distribution  $g(x)$ , rather than the heat flux, specified on the collector-base junction. From Eq. (2), we find at  $y = 0$  that

$$T_{y=0} - T_s = \frac{a_0}{2}b + \sum_{n=1}^{\infty} a_n \left( \frac{a}{n\pi} \right) \cos \frac{n\pi x}{a} \tanh \frac{bn\pi}{a} \quad (5)$$

We now define

$$b_n = a_n \left( \frac{a}{n\pi} \right) \tanh \frac{bn\pi}{a} \quad \text{and} \quad b_0 = a_0 b \quad (6)$$

Substituting these into Eq. (5), we obtain

$$T_{y=0} - T_s = \frac{b_0}{2} + \sum_{n=1}^{\infty} b_n \cos \frac{n\pi x}{a} = g(x) \quad (7)$$

Hence,

$$b_n = \frac{a}{2} \int_0^a g(x) \cos \frac{n\pi x}{a} dx \quad (8)$$

and

$$T - T_s = \frac{b_o}{2} \left(1 - \frac{y}{b}\right) + \sum_{n=1}^{\infty} b_n \cos \frac{n\pi x}{a} \sinh \frac{(b-y)n\pi}{a} \operatorname{cosech} \frac{bn\pi}{a} \quad (9)$$

Suppose that  $g(x)$  (the temperature at  $y = 0$ ) were measured in some way. From this,  $f(x)$  may be found by using Eqs. (3), (6), and (8), to obtain

$$f(x) = \frac{a}{4b} \int_0^a g(x) dx + \sum_{n=1}^{\infty} \left(\frac{n\pi}{2}\right) \cos \frac{n\pi x}{a} \coth \frac{bn\pi}{a} \int_0^a g(x) \cos \frac{n\pi x}{a} dx \quad (10)$$

If it is assumed that the surface of the transistor is at  $g(x)$ , then the temperature could conceivably be measured by using a series of materials with progressively increasing melting points. Each material would be coated in turn onto the device and the position of melting observed.

#### C. RELATION BETWEEN CURRENT DISTRIBUTION AND HEAT GENERATION

Once  $f(x)$  is obtained, perhaps in the manner indicated in the previous section, it should be possible to say something about the distribution of current under the emitter. Conversely, if the current distribution is known, it should then be possible to arrive at  $f(x)$ .

It is assumed that the heat generation at any point under the emitter is proportional to the current density  $j(x)$  at that point, or that

$$j(x) = \frac{f(x)}{K} \quad (11)$$

where  $K$  is a constant that will be determined. This is of course only a first-order approximation;  $K$  would also be a weak function of the transistor operating conditions.

If the total current per unit emitter length is  $i$ , then

$$i = 2 \int_0^a j(x) dx = 2 \int_0^a \frac{f(x)}{K} dx \quad (12)$$

Hence,

$$K = 2 \int_0^a \frac{f(x)}{i} dx$$

and

$$j(x) = \frac{i f(x)}{2 \int_0^a f(x) dx} \quad (13)$$

In practice, however, a heavy metallic contact layer lies over the emitter and tends to even out the temperature of the emitter and invalidate Eq. (11).

### III. EXAMPLES

#### A. SEMI-INFINITE ONE-DIMENSIONAL TRANSISTOR

Fletcher has given an approximate solution for the current distribution in an idealized semi-infinite one-dimensional transistor.<sup>1</sup> The solution is of the form

$$j(x) = j_0 [1 + Ax_e]^{-2} \quad (14)$$

<sup>1</sup>N.H. Fletcher, "Some Aspects of the Design of Power Transistors," Proc. IRE 43, 551-59(1955).

where  $A$  is a constant and  $x_e$  is the distance in from the edge of the emitter,  $w - x$ . This expression tells us that the current is concentrated near the edges of the emitters. Unfortunately, when this equation is substituted into Eq. (13) and this in turn substituted into Eq. (4), the result cannot be integrated analytically.

Thus, we are reminded that, if analytical results are desired,  $f(x)$  must be of such a form that  $f(x) \cos n\pi x/a$  can be integrated. Furthermore, because the current is known to concentrate near the edges of the emitters,  $f(x)$  must be constant or continually increasing with  $x$ . Some functions that fulfill these two conditions are:  $A$ ,  $Ax^s$  ( $s=1, 2, \dots$ ),  $A \sin Bx$ ,  $Ae^{Bx}$ , and any sums thereof. Because of its simplicity, only the application of the first function will be considered in detail here.

#### B. UNIFORM HEAT GENERATION

We consider now the case of uniform heat generation beneath an emitter of width  $2w$ . Let  $q$  be the rate of heat generation per unit length of emitter (e.g., watt/cm). At the collector-base junction  $y = 0$ , we then have

$$-k \frac{\partial T}{\partial y} = \frac{q}{2w} \quad \text{or} \quad -\frac{\partial T}{\partial y} = \frac{q}{2kw}$$

Hence

$$\begin{aligned} f(x) &= \frac{q}{2kw} & \text{for} & \quad 0 \leq x \leq w \\ f(x) &= 0 & \text{for} & \quad w < x \leq a \end{aligned} \quad (15)$$

By substituting this into Eq. (4), we obtain

$$\begin{aligned} a_n &= \frac{2}{a} \int_0^w \left( \frac{q}{2kw} \right) \cos \frac{n\pi x}{a} dx \\ &= \frac{q}{n\pi kw} \sin \frac{n\pi w}{a} \end{aligned} \quad (16)$$

and

$$a_o = \frac{2}{a} \int_0^w \left( \frac{q}{2kw} \right) dx = \frac{q}{ak} \quad (17)$$

Substitution of (16) and (17) into Eq. (2) then yields the desired solution for uniform heat generation under the emitters,

$$T - T_s = \frac{q}{2ak}(b - y) + \sum_{n=1}^{\infty} \left( \frac{qa}{n^2 \pi^2 kw} \right) \sin \frac{n\pi w}{a} \cos \frac{n\pi x}{a} \sinh \frac{(b - y)n\pi}{a} \operatorname{sech} \frac{bn\pi}{a} \quad (18)$$

In the actual operation of the transistor, we would like to know or to control the maximum temperature  $T_m$ . This clearly will be reached at  $y = 0$ . For arbitrary  $f(x)$ , the only general way to find the  $x$ -coordinate for the maximum temperature is to calculate and plot

$$T_{y=0}(x) = g(x) = \frac{b}{a} \int_0^a f(x) dx + \sum_{n=1}^{\infty} \left( \frac{2}{n\pi} \right) \cos \frac{n\pi x}{a} \tanh \frac{bn\pi}{a} \int_0^a f(x) \cos \frac{n\pi x}{a} dx \quad (19)$$

In the present case of uniform heat generation, it is apparent that  $T_m$  will be at  $x = 0$ , under the center of the emitter.

If we define  $\phi = (T_m - T_s)/(q/k)$  as a dimensionless maximum temperature, then the desired result is obtained by setting  $x = 0$ ,  $y = 0$  in Eq. (18) to obtain

$$\phi = \frac{1}{2} \left( \frac{b}{a} \right) + \frac{1}{(w/a)\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin n\pi \left( \frac{w}{a} \right) \tanh n\pi \left( \frac{b}{a} \right) = \phi \left( \frac{w}{a}, \frac{b}{a} \right) \quad (20)$$

Plots of calculations based on this result are shown in Figs. 3, 4, and 5. It should be noted that the series converges slowly for small  $w/a$ , 25 terms being necessary for  $w/a = 0.1$ .

If the emitter occupies the whole area of  $y = 0$ , which of course is not a practical device, then  $w/a = 1$  and  $\sin n\pi(w/a) = 0$ . For this case,  $\phi = (1/2)(b/a)$ , which is the solution for uniform heat flow between an infinite parallel source and sink. We call this  $\phi_1(b/a)$ . For  $b/a > 1$ ,  $\tanh \pi(b/a) \approx 1$ , and

$$\phi = \frac{T_m - T_s}{q/k} \approx \phi_1\left(\frac{b}{a}\right) + \phi_2\left(\frac{w}{a}\right) \quad (21)$$

where

$$\phi_2\left(\frac{w}{a}\right) = \frac{1}{(w/a)\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin n\pi\left(\frac{w}{a}\right)$$

This means that, for a large device thickness  $b$ , the solution is the uniform heat flow solution plus a function of the ratio of the emitter width to emitter spacing  $w/a$ . This fact is illustrated by the straight line portions of the curves given in Fig. 3.

A parameter of interest to transistor designers in the past has been the distance at which a neighboring emitter first affects the thermal behavior of the emitter region under consideration. If we consider the emitter width  $2w$  and device thickness  $b$  to be fixed, then the variation in maximum temperature, as given by  $\phi$ , is a function of emitter spacing, as shown in Fig. 5. As  $a/b$  decreases, the emitter spacing grows smaller. It is seen that the interference between emitters has effectively vanished for  $a/b > 2.5$ . This suggests that we can use the solution given by Eq. (18) for the isolated emitter solution desired in Section I, if we only set  $a/b > 2.5$ . The value of  $a/b$  should not be set too high because, as mentioned previously, this

leads to slow convergence of the solution. Consequently, a value of 3 for  $a/b$  was chosen as a compromise to give, at  $y = 0$  for an isolated emitter,

$$\frac{T_{y=0} - T_s}{q/k} = \frac{1}{6} + \frac{3}{\pi^2(w/b)} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{3} n\pi \left(\frac{w}{b}\right) \cos \frac{1}{3} n\pi \left(\frac{x}{b}\right) \tanh \frac{n\pi}{3} \quad (22)$$

For a series of  $n$  emitters spaced  $2a$  apart, we must add the solutions for each, substituting  $x \pm j2a$  for  $x$  in the solution for each particular emitter. Here  $j$  is the integral number of spacings  $2a$  by which each particular emitter is to the left or right ( $\pm$ ) of the emitter under consideration (that is, the one at  $x = 0$ ). If we perform this summation and solve for the temperature at the center  $x = 0$  of the emitter under consideration, we obtain

$$\phi = \frac{m}{b} + \frac{b}{\pi^2(w/b)} \sum_j \sum_n \frac{1}{n^2} \sin \frac{1}{3} n\pi \left(\frac{w}{b}\right) \cos \frac{2}{3} n\pi j \left(\frac{a}{b}\right) \tanh \frac{n\pi}{3} \quad (23)$$

where

$$j = 0, \pm 1, \dots$$

We return now to the original problem of an infinite number of emitters. In designing a power transistor from a thermal standpoint, it is desired to maximize the total current  $I$  of the device without exceeding a maximum temperature  $T_m$  anywhere within the device. If we consider a device of width  $L$ , then the total number of emitters present is  $L/2a$ . Hence, the current in any single emitter is  $i = I/(L/2a)$ . If it is assumed that the heat generated beneath any emitter is  $q = iV$ , then

$$q = 2IaV/L$$

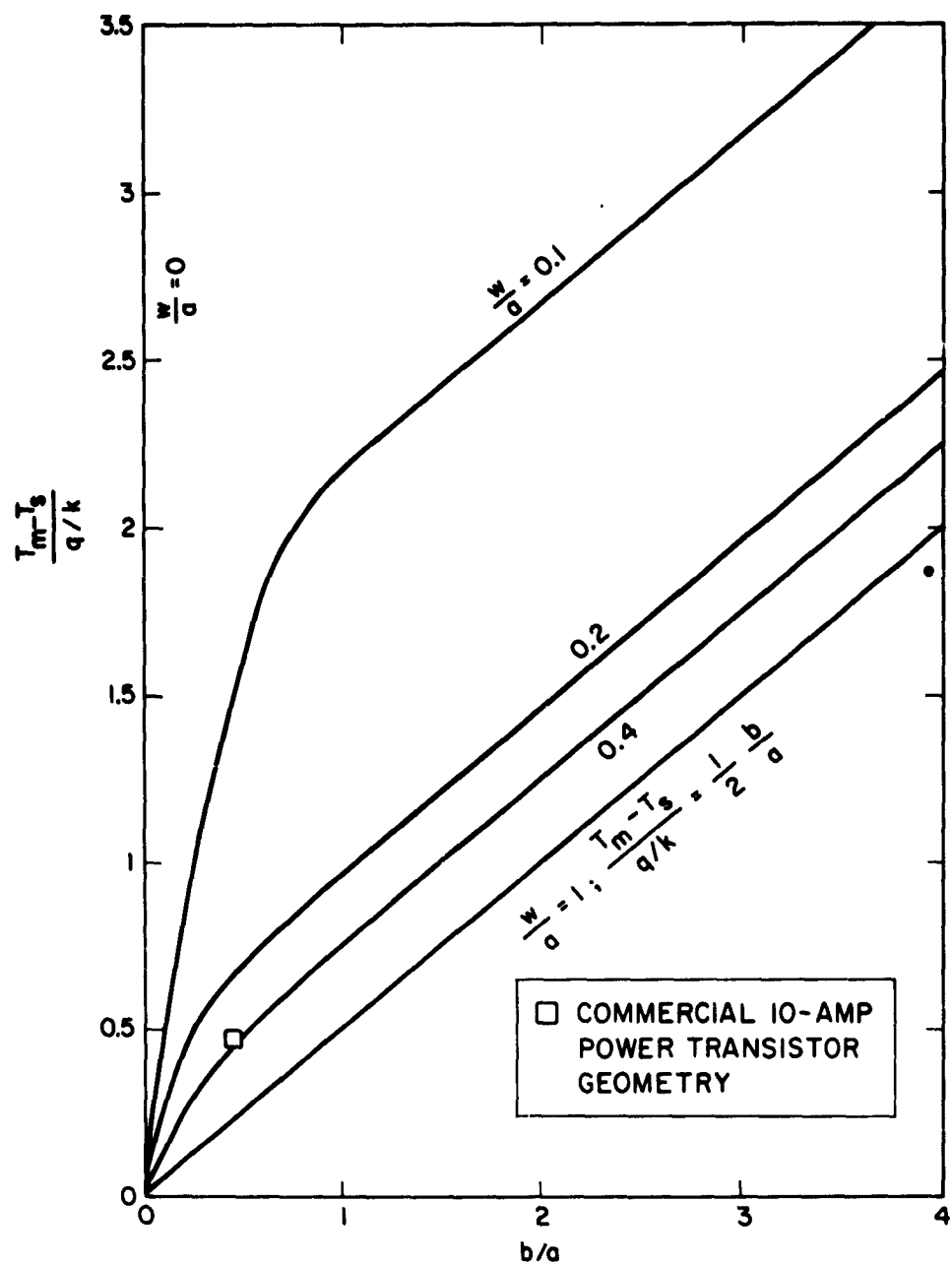


Fig. 3. Maximum Temperature in Power Transistor as a Function of Ratio of Device Thickness to Half of Emitter Spacing; Heat Assumed to be Liberated Uniformly Under Emitter



But, from the definition of  $\phi$ ,

$$q = \frac{k(T_m - T_s)}{\phi}$$

therefore,

$$I = \frac{Lk(T_m - T_s)}{2aV\phi} \quad (24)$$

The maximum total device current results when  $1/a\phi$  is maximum. If, for example,  $b$  and  $w/a$  are regarded as known, then this will be at the maximum of  $(b/a)/b\phi$ . With a constant  $\gamma$  defined as

$$\gamma = \frac{Lk(T_m - T_s)}{2bV}$$

then Eq. (24) becomes

$$I = \frac{\gamma(b/a)}{\phi}$$

The maximum  $I$  then occurs at

$$\frac{\partial I}{\partial (b/a)} = \gamma \left[ \phi^{-1} - \left( \frac{b}{a} \right) \phi^{-2} \frac{\partial \phi}{\partial (b/a)} \right] = 0 \quad (25)$$

or

$$1 = \frac{b/a}{\phi} \frac{\partial \phi}{\partial (b/a)} = \frac{\partial \ln \phi}{\partial \ln (b/a)} \quad (26)$$

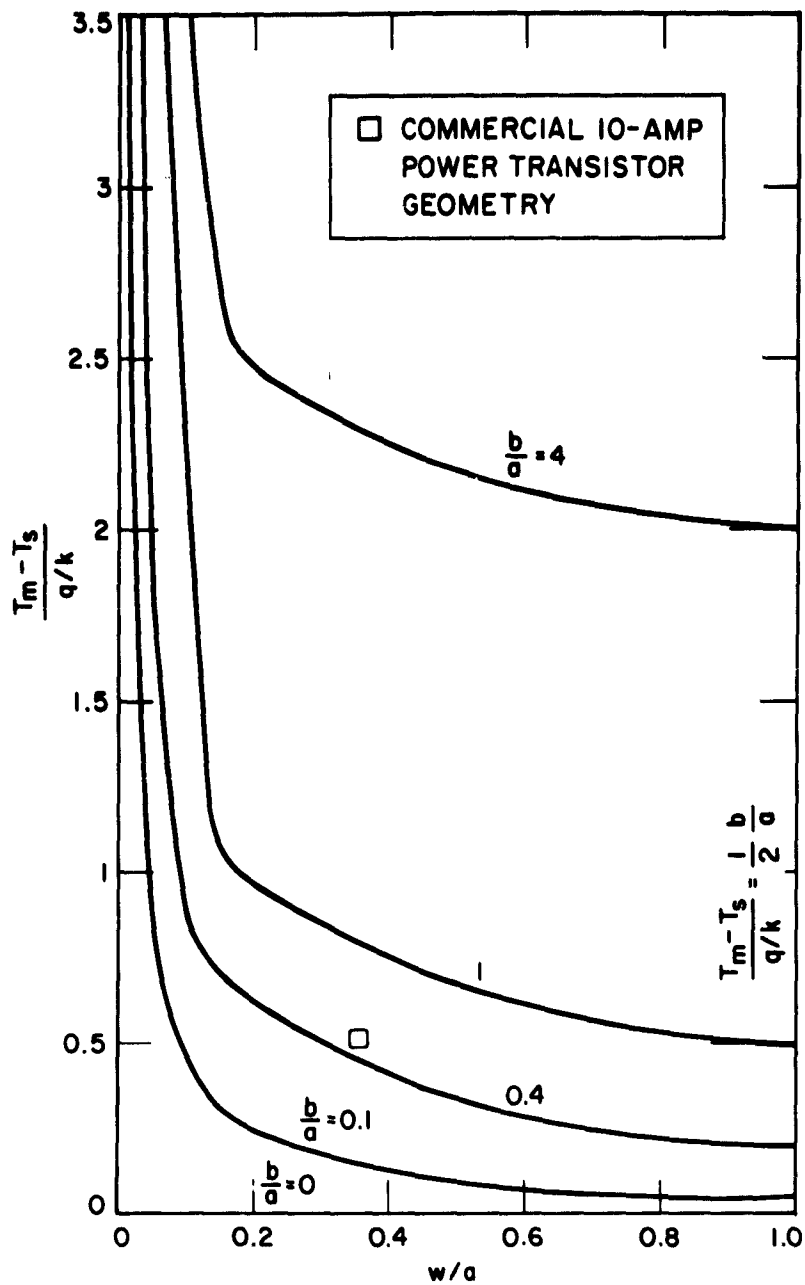


Fig. 4. Maximum Temperature in Power Transistor as a Function of Ratio of Emitter Width to Emitter Spacing; Heat Assumed to be Liberated Uniformly Under Emitter

In other words, the maximum current would be at the point on a plot of  $\ln \phi$  vs  $\ln (b/a)$  where the slope is 1. As might be expected, no such maximum occurs for the present heat distribution. The best heat dissipation will be obtained when  $w/a = 1$ , which is no longer a practical transistor. Having a minimum possible device thickness  $b$  will also give the maximum heat dissipation.

The foregoing results may also be reached in a more intuitive manner as follows: By examination of Fig. 5, it is seen that the value of  $(T_m - T_g)/(q/k)$  increases as the emitter spacing is increased. On the other hand, more emitters can be placed in the same space for closer emitter spacings. Hence the maximum heat input to the device would be given by the maximum of  $q/a$ . This will occur at the minimum of  $\phi(a/b)$  for constant  $w/b$ . If this operation is performed, it is again found that the optimum occurs at  $w = a$ , for which the emitters are all touching.

We should note at this time that, in the preceding analysis, we have assumed  $T_g$  to be a constant, independent of the total device heat flux. This is not true, however, because the device itself has thermal resistance to the flow of heat to the surroundings, and  $T_g$  will increase with the total device heat flux in an approximately linear fashion. Nevertheless, this fact will not change the above conclusions. •

For non-uniform heat liberation in the emitter, however, it should be possible to obtain an optimum device geometry. In any case, it is clear that the final design will not be dictated solely by thermal considerations. Electrical performance characteristics and available fabrication techniques are prime determining factors in any design. The effects of the inevitable design compromises are shown as the squares in Figs. 3, 4, and 5, which represent the geometry of a commercial 10-amp power transistor.

#### C. EMITTER PARTLY EFFECTIVE

In this section, we consider the case in which the current is assumed to flow through only the outside section  $w_e$  of the emitter, but uniformly in that portion. This condition is expressed by

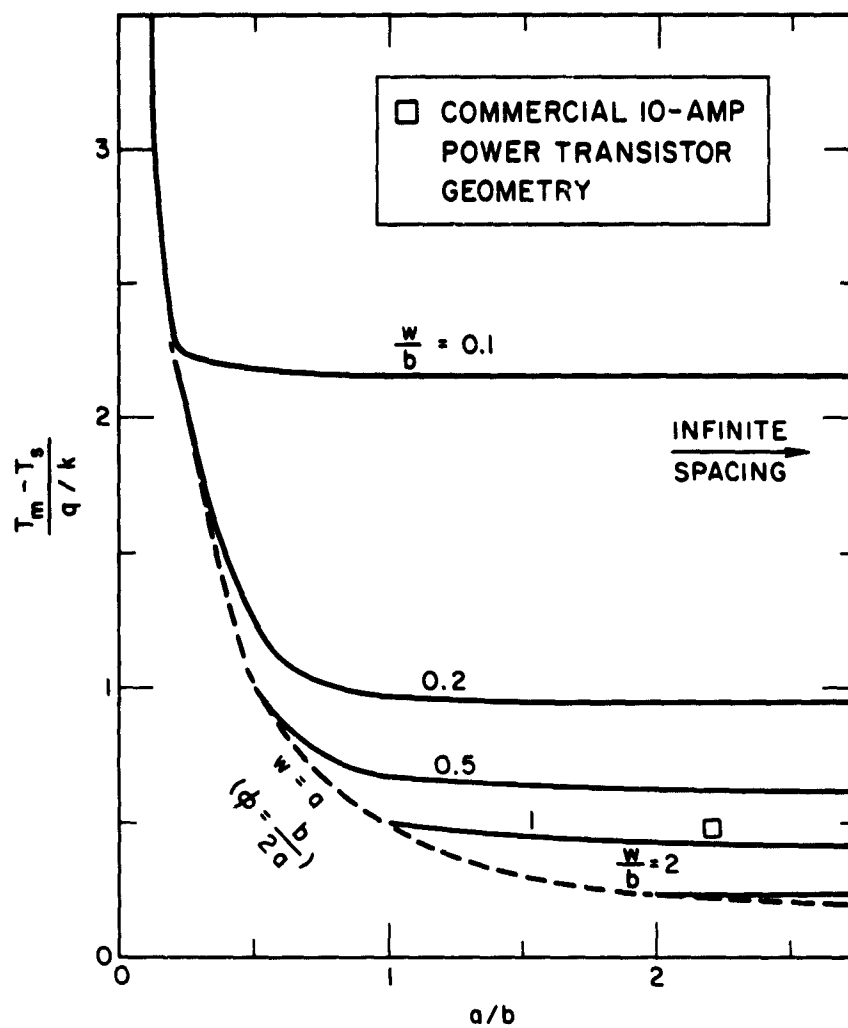


Fig. 5. Maximum Temperature in Power Transistor as a Function of Ratio of Half of Emitter Spacing to Device Thickness; Heat Assumed to be Liberated Uniformly Under Emitter

$$f(x) = \frac{q}{2kw_e} \quad \text{for} \quad w - w_e \leq x \leq w$$

$$f(x) = 0 \quad \text{for} \quad 0 \leq x < w - w_e \text{ and } w < x \leq a$$

Substitution of this condition into Eq. (2) and Eq. (4) yields

$$\begin{aligned} \frac{T - T_s}{q/k} = \frac{b - y}{2a} + \frac{1}{(w_e/a)\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[ \sin n\pi \left( \frac{w}{a} \right) - \sin n\pi \left( \frac{w - w_e}{a} \right) \right] \\ \times \cos n\pi \left( \frac{x}{a} \right) \sinh n\pi \left( \frac{b - y}{a} \right) \operatorname{sech} n\pi \left( \frac{b}{a} \right) \end{aligned} \quad (27)$$

We will leave detailed examination of this result to the reader.

#### D. OTHER HEAT DISTRIBUTIONS

Solutions for other current-heat source distributions may likewise be easily found by substitution of the respective functions  $f(x)$  into Eq. (2) and Eq. (4). The results may be quite complex, but the principles are identical to those outlined above for the most elementary case.

#### IV. CONCLUSION

The internal heat transfer problem for an interdigitated power-transistor structure has been solved analytically. The relations among current distribution, heat generation, and temperature distribution have been shown. Application of the results to the most elementary case illustrates usage of the equations. In conjunction with considerations of other device characteristics and current fabrication techniques, these heat transfer results should permit a more optimum design of power transistors.

## NOMENCLATURE

$A$	constant
$a$	one-half of emitter spacing, cm
$a_n$	$n$ th constant in solution, given in Eq. (4)
$B$	constant
$b$	thickness of collector region, cm
$b_n$	Fourier series constant defined by Eqs. (6) and (8)
$f(x)$	$1/k$ of heat liberation rate at any point $x$ under emitter, $^{\circ}\text{C}/\text{cm}$
$g(x)$	temperature ( $^{\circ}\text{C}$ ) at any point $x$ on $y = 0$
$I$	total current of all emitters per unit length of single emitter, amp/cm
$i$	total current for one emitter per unit length of emitter, amp/cm
$j(x)$	current density at any point on emitter, $\text{amp}/\text{cm}^2$
$j_0$	constant in Eq. (14), $\text{amp}/\text{cm}^2$
$K$	constant in Eq. (11)
$k$	thermal conductivity, $\text{watt}/\text{cm } ^{\circ}\text{C}$
$L$	length of device, cm
$m$	total number of emitters (Eq. 23)
$n$	index, 0, 1, 2, ...
$q$	heat liberated at each emitter per unit length of emitter, $\text{watt}/\text{cm}$
$s$	integral constant, 0, 1, 2, ...
$T$	temperature at $(x, y)$ , $^{\circ}\text{C}$
$T_m$	maximum temperature in transistor, $^{\circ}\text{C}$

# NOMENCLATURE (Continued)

$T_s$	temperature of heat sink at collector, °C
$V$	effective potential drop for current flow in volts, so that $q = iV$
$w$	one half of emitter width, cm
$w_e$	one half of effective emitter width, from which current flows, cm
$x$	distance from center plane of emitter, cm (see Fig. 2)
$x_e$	distance from outside edge of emitter, cm ( $w - x$ )
$y$	distance from top surface, cm (see Fig. 2)
$\gamma$	$\frac{Lk(T_m - T_s)}{2bV}$
$\phi$	$\frac{T_m - T_s}{q/k}$
$\phi_1$	$\frac{1}{2}\left(\frac{b}{a}\right)$ , ratio of device width to emitter spacing
$\phi_2$	$\frac{1}{(w/a)\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin n\pi \left(\frac{w}{a}\right)$
$w/a$	ratio of emitter width to emitter spacing
$b/a$	twice ratio of device width to emitter spacing
$\sinh x$	$\frac{e^x - e^{-x}}{2}$
$\cosh x$	$\frac{e^x + e^{-x}}{2}$
$\operatorname{sech} x$	$1/\cosh x$

NOMENCLATURE (Continued)

$\operatorname{cosech} x$        $1/\sinh x$

$\tanh x$            $\sinh x/\cosh x$

$\coth x$            $\cosh x/\sinh x = 1/\tanh x$



#### ACKNOWLEDGEMENT

The suggestion of this problem by G. Otto is gratefully acknowledged. We wish to thank F. Steinebrey and J. Buie of Pacific Semiconductors, Inc. for their helpful discussions.

UNCLASSIFIED	<p>Aerospace Corporation, El Segundo, California. HEAT TRANSFER IN POWER TRANSISTORS. Prepared by W. R. Wilcox. 15 February 1963. [28] p. incl. illus. (Report TDR-169(3240-10)TR-3;SSD-TDR-63-39) (Contract AF 04(695)-169) Unclassified report</p> <p>The internal heat transfer problem for a typical power-transistor structure has been solved analytically. The relations among current distribution, heat generation, and temperature distribution have been derived. Usage of the resulting equations is illustrated by application to the most elementary problem, namely, uniform heat generation under the emitter.</p>
UNCLASSIFIED	UNCLASSIFIED

UNCLASSIFIED	<p>Aerospace Corporation, El Segundo, California. HEAT TRANSFER IN POWER TRANSISTORS. Prepared by W. R. Wilcox. 15 February 1963. [28] p. incl. illus. (Report TDR-169(3240-10)TR-3;SSD-TDR-63-39) (Contract AF 04(695)-169) Unclassified report</p> <p>The internal heat transfer problem for a typical power-transistor structure has been solved analytically. The relations among current distribution, heat generation, and temperature distribution have been derived. Usage of the resulting equations is illustrated by application to the most elementary problem, namely, uniform heat generation under the emitter.</p>
UNCLASSIFIED	UNCLASSIFIED

UNCLASSIFIED	<p>Aerospace Corporation, El Segundo, California. HEAT TRANSFER IN POWER TRANSISTORS. Prepared by W. R. Wilcox. 15 February 1963. [28] p. incl. illus. (Report TDR-169(3240-10)TR-3;SSD-TDR-63-39) (Contract AF 04(695)-169) Unclassified report</p> <p>The internal heat transfer problem for a typical power-transistor structure has been solved analytically. The relations among current distribution, heat generation, and temperature distribution have been derived. Usage of the resulting equations is illustrated by application to the most elementary problem, namely, uniform heat generation under the emitter.</p>
UNCLASSIFIED	UNCLASSIFIED

UNCLASSIFIED	<p>Aerospace Corporation, El Segundo, California. HEAT TRANSFER IN POWER TRANSISTORS. Prepared by W. R. Wilcox. 15 February 1963. [28] p. incl. illus. (Report TDR-169(3240-10)TR-3;SSD-TDR-63-39) (Contract AF 04(695)-169) Unclassified report</p> <p>The internal heat transfer problem for a typical power-transistor structure has been solved analytically. The relations among current distribution, heat generation, and temperature distribution have been derived. Usage of the resulting equations is illustrated by application to the most elementary problem, namely, uniform heat generation under the emitter.</p>
UNCLASSIFIED	UNCLASSIFIED